Natural convection flow of non-Newtonian power-law fluid from a slotted vertical isothermal surface

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Abstract

Purpose – The paper's aim is to investigate the natural convection flow of an Ostwald-de Waele type power law non-Newtonian fluid past an isothermal vertical slotted surface.

Design/methodology/approach – The Keller-Box method is used to solve the governing boundary layer equations for the natural convection flow of an Ostwald-de Waele type power law non-Newtonian fluid past an isothermal vertical slotted surface.

Findings – As the slip parameter increases, the friction factor increases whereas the heat transfer rate decreases. Owing to increase in the value of the Prandtl number, Pr, there is decrease in the value of the skin-friction coefficient, and augmentation of heat transfer rate. As the viscosity index n increases, both the friction factor and the heat transfer rate increase.

Research limitations/implications – The analysis is valid for steady, two-dimensional laminar flow of an Ostwald-de Waele type power law non-Newtonian fluid past an isothermal vertical slotted surface. An extension to three-dimensional flow case is left for future work.

Practical implications – The method is useful to analyze perforated plates and wire netting such as perforated wings in order to reduce the drag by suction of the boundary layer, filtration or air-conditioning.

Originality/value – The results of this study may be of interest to engineers interested in heat transfer augmentation and drag reduction in heat exchangers.

Keywords Convection, Fluids, Surface texture, Boundary layers

Paper type Research paper

Nomenclature

C	skin-friction coefficient
e_{ij}	strain components
f	non-dimensional, reduced stream function
F	non-dimensional stream function f expressed in terms of η
G	non-dimensional, reduced temperature function
Gr_x	local Grashof number
k	thermal conductivity of the fluid
K	power-law fluid viscosity parameter
C_p	specific heat at constant pressure
m	surface temperature gradient
n	power law index
Nu_x	local Nusselt number
Pr	Prandtl number ($\nu\alpha$)
Re_x	local Reynolds number ($U_{ref}\,x/\,\nu$)
T	temperature of the fluid

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T_w	surface temperature of the slotted surface
T_r	reference temperature
T_∞	ambient temperature
u, v	velocity components along the x, y axes, respectively
U_{ref}	reference velocity
x, y	Cartesian coordinate measured along the surface and normal to it, respectively

Greek symbols

α	thermal diffusivity
β	coefficient of thermal expansion
ρ	fluid density
δ_{ij}	Kroneckar delta
λ	velocity slip parameter
η	transformed spatial variable
μ	dynamic viscosity
ν	(K/ρ)
ψ	non-dimensional stream function

Introduction

Natural convection in a laminar boundary layer flow has been analyzed extensively for semi-infinite flat plates in vertical, horizontal, and inclined orientations. Typical studies can be found in the works of Ostrach (1996), Pera and Gebhart (1973), Hasan and Eichorn (1982), Sparrow and Gregg (1958) and Semenv (1984). Keller and Yang (1972) employed a Gortler-type series to study the free convection boundary layer over a non-isothermal vertical plate and in their analysis the wall temperature was assumed to be represented by a power series in the streamwise coordinate. Kao *et al.* (1977) proposed the method of strained coordinates for the computation of the wall heat transfer parameter for a plate with an arbitrary prescribed surface temperature. In this method, the coordinate along the plate was transformed by using an integral function of specified wall temperature so that the problem can be solved once for all with any specified surface conditions. Further Yang *et al.* (1982) proposed an alternative method to evaluate the surface heat transfer rate and the wall shear stress for this vertical free convection boundary layer flow considered by Kao *et al.* (1977) using Mark-type series solution (Merk, 1959). The governing coupled partial differential equations were transformed into a sequence of coupled ordinary differential equations, which were then solved numerically.

Perforated plates and wire netting occur in many applications of fluid mechanics (such as perforated wings in order to reduce the turbulence by suction of the boundary layer, filtration, or air-conditioning). These media are often characterized by their pressure drop coefficient which is mostly determined experimentally. When faced with a tangential flow, no-slip hypothesis is generally taken and this assumption is no longer valid when the perforation density is sufficiently large. In a boundary layer flow this will have consequence for the displacement thickness. Working on an ideal two-dimensional periodic perforated media placed in a purely tangential flow it has been proved by Laplace and Arquis (1998) that the shear stress at the wall is equal to the slip velocity times a coefficient λ_0, applies to slotted plate. By means of an experimental study, this slip condition (called the Navier condition hereafter) was revived empirically by Beavers and Joseph (1967) for a fluid-porous medium interface. A theoretical justification for it was given by Saffman (1971).

All the above mentioned papers were concerned with the analyses of Newtonian fluids. It is well known that a number of industrial fluids such as molten plastics, polymers, food stuffs, or slurries exhibit non-Newtonian fluid behavior. Therefore a study of heat and mass transfer in non-Newtonian fluids is also of practical importance. Visco-elastic fluids, couple stress fluids, micropolar fluids, power-law fluids are a few different types of non-Newtonian fluids. The simplest and most common type is the power-law fluid (Ostwald-de Waele model) for which the rheological equation of the state between the stress components τ_{ij} and strain components e_{ij} is defined by Vujanovic *et al.* (1972)

$$\tau_{ij} = -p\ \delta_{ij} + K\left|\sum_{m=1}^{3}\sum_{l=1}^{3} e_{lm}\ e_{lm}\right|^{\frac{n-1}{2}} e_{ij}.$$

where p is the pressure, δ_{ij} is Kroneckar delta and K and n are the consistency coefficient and power-law index of the fluid. For $n > 1$, fluid is said to be dilatant or shear thickening; for $n < 1$ the fluid is called shear thinning or pseudoplastic and for $n = 1$, the fluid is simply the Newtonian fluid. Several fluids studied in the literature suggest the range $0 < n \leq 2$ for the power law index n. Considerable amount of research work has been done in this field by taking into account of heat and mass transfer. Schowalter 1960 studies the boundary layer flow in pseudo plastic fluids ($n < 1$). Lee and Ames (1966) extended this work to find the similarity solutions for non-Newtonian power-law fluid. Acrivos (1960) and Kawase and Ulbrecht (1984) provided an approximate analysis for the free convection of non-Newtonian fluids over a flat plate. Huang *et al.* (1989) presented similarity solutions for free convection from a vertical plate to power law non-Newtonian fluids.

The objective of the present paper is to investigate the natural convection flow of a power law non-Newtonian fluid past a slotted vertical surface. The equations that govern the flow and heat transfer are reduced to local non-similarity form. The transformed boundary layer equations are solved numerically using implicit finite difference method together with Keller-box elimination technique (Keller, 1978) for all values of ξ in the interval [0.0, ∞]. The solutions for a rigid surface are compared with literature values. The results are expressed in terms of the local skin-friction and local rate of heat transfer coefficients against ξ for different Prandtl number, Pr, and the slip velocity coefficient λ_0.

Formulation of the problem

Consider the natural convection flow of power-law non-Newtonian fluid from a slotted isothermal vertical surface, as shown in the schematic diagram Figure 1. The x coordinate is measured along the surface from the point where the surface originates and the y coordinate is measured normal to it.

The governing equations for steady, laminar, two-dimensional and incompressible viscous flow of a non-Newtonian fluid with constant physical properties can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial}{\partial y}\left[\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right] + g\beta(T - T_\infty) \tag{2}$$

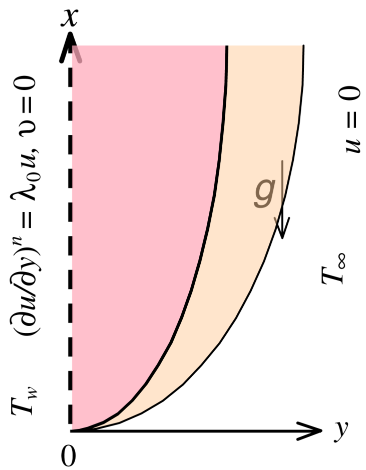

Figure 1.
The flow configuration and the coordinate

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

Boundary conditions

$$\left(\frac{\partial u}{\partial y}\right)^n = \lambda u, \quad v = 0,\ T = T_w \quad \text{at } y = 0 \quad u = 0,\ T = T_\infty \quad \text{at } y = 0 \tag{4}$$

In equation (2) n is a positive real number or the flow index that designates the nature of the fluid. When n =1, the equation governs the flow of a Newtonian fluid. ν is generalized kinematic viscosity, β the volume expansion coefficient, and α is the thermal diffusivity. Further, in the boundary condition (4) λ is defined as slip constant. It can be used to express the boundary behavior of total adhesion ($\lambda = +\infty$ and therefore $u = 0$) and of total slip ($\lambda = 0$ and that implies $\partial u/\partial y = 0$). This has been proved by Laplace and Arquis (1998) that, this partial slip boundary condition applies to slotted surface.

Due to the nature of the non-Newtonian power-law fluid, similarity solutions only exist at infinite Prandtl number; i.e., when inertia terms in equation (2) be neglected. But when the inertia force becomes of the same order of magnitude as the viscous force, similarity solutions do not exist. Therefore, using the dimensional analysis, one can obtain the following similarity transformations in order to facilitate the solutions of equations (1)-(4) reducing to non-similar equations. So here we introduce the following group of transformations:

$$\begin{gathered}\psi(x,y) = \left[\nu x (xg\beta\Delta T)^{(2n-1)/2}\right]^{1/(1+n)} f(\xi,\eta), \quad \frac{T(x,y) - T_\infty}{\Delta T} = g \\ \xi = \frac{x}{L}, \quad \eta = \left[(xg\beta\Delta T)^{(2-n)/2} x^n/\nu\right]^{1/(1+n)} \frac{y}{x}\end{gathered} \tag{5}$$

where, $\Delta T = T_0 x^m$, m being the gradient of surface temperature, L is their reference length and ψ is the stream function which is defined so that it satisfies the continuity equation:

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \tag{6}$$

Substitutions of the transformations (5) and the definition (6) in equations (2) and (3) yield following locally non-similar equations:

$$\left[f''^{n-1}f''\right]' + \frac{(1+2n)}{2(1+n)}ff'' - \frac{1}{2}f'^2 + g = \xi\left(f'\frac{\partial f'}{\partial \xi} + f''\frac{\partial f}{\partial \xi}\right) \tag{7}$$

$$\mathrm{Pr}^{-1}\xi^{\frac{(1-n)}{2(1+n)}}g'' + \frac{(1+2n)}{2(1+n)}fg' = \xi\left(f'\frac{\partial g}{\partial \xi} + g'\frac{\partial f}{\partial \xi}\right) \tag{8}$$

The boundary conditions to be satisfied by the above equations are

$$\begin{gathered} f(\xi,0) = 0, \quad f''(\xi,0) = \lambda\xi^{1/2(1+n)}\left[f'(\xi,0)\right]^{1/n}, \quad g(\xi,0) = 1 \\ f'(\xi,\infty) = 0, \quad g(\xi,\infty) = 0 \end{gathered} \tag{9}$$

In equation (8) we have

$$\mathrm{Pr} = \frac{\nu^{2/(1+n)}}{\alpha}\left(g\beta T_0 L^3\right)^{\frac{3(n-1)}{2(1+n)}} \tag{10}$$

Once we know the solutions of the set of equations (7) and (8) satisfying the condition (9) we can express the local heat transfer, in terms of local Nusselt number, from the relation given below:

$$Nu_x Gr_x^{-1/2(1+n)} = -g'(\xi,0) \tag{11}$$

where Gr_x is the local Grashof number, defined as

$$Gr_x = \frac{(g\beta\Delta T)^{2-n}x^{2+n}}{\nu^2} \tag{12}$$

In the present investigation, consideration of $n = 1$ (when the fluid is Newtonian) reduces to that investigated.

We now proceed to integrate the locally non-similar partial differential equations (7) and (8) subject to the boundary conditions (9) using the implicit finite difference method. The partial differential equations (7) and (8) are first converted into a system of first order equations with dependent variables $u(\xi, \eta)$, $v(\xi, \eta)$, and $p(\xi, \eta)$ as follows:

$$f' = U, \quad U' = V, \quad g' = P \tag{13}$$

Equations (7) and (8) then take the forms:

$$V' + p_1 fV - p_2 U^2 + \theta = \xi\left[U\frac{\partial U}{\partial \xi} - V\frac{\partial f}{\partial \xi}\right] \tag{14}$$

$$PR(\xi)P' + p_1 fP = \xi\left[U\frac{\partial g}{\partial \xi} - P\frac{\partial f}{\partial \xi}\right] \tag{15}$$

$$f = 0, \quad V = p_3(\xi)U^n, \quad \theta = 1 \text{ at } \eta = 0 \quad U = 0, \quad g = 0 \quad \text{as} \quad \eta \to \infty \tag{16}$$

where

$$p_1 = \frac{1 = 2n}{2(1+n)}, \quad p_2 = \frac{1}{2}, \quad p_3(\xi) = \lambda\xi^{\frac{1}{2n(1+n)}}, \quad PR(\xi) = \Pr^{-1}\xi^{\frac{1-n}{2(1+n)}} \tag{17}$$

We now consider the net rectangle on the (ξ, η) plane and denote the net points by

$$\xi^0 = 0, \ \xi^i = \xi^{i-1} + k_i \quad i = 1, 2, \ldots, \mathrm{N} \tag{18}$$

$$\eta_0 = 0, \ \eta_j = \eta_{j-1} + l_j \quad j = 1, 2, \ldots, J, \ \eta_j = \eta_\infty \tag{19}$$

Here i and j index points on the (ξ, η) plane, and k_i and l_j give the variable mesh width.

We approximate the quantities (f, U, V, g, P) at points (ξ^l,η_j) of the net by (f_j^i, U_j^i, V_j^i, g_j^i, P_j^i) which we call the net function. The notation m_j^i is also employed for any net function quantities midway between the net points as follows:

$$\xi^{i-1/2} = \frac{1}{2}\left(\xi^i + \xi^{i-1}\right), \eta_{j-1/2} = \frac{1}{2}\left(\eta_j + \eta_{j-1}\right) \tag{20}$$

$$m_j^{i-1/2} = \frac{1}{2}\left(m_j^i + m_j^{i-1}\right), m_{j-1/2}^i = \frac{1}{2}\left(m_j^i + m_{j-1}^i\right) \tag{21}$$

We now write the difference equations that are to approximate equations (2.59)-(2.62) by considering one mesh rectangle. We start by writing the finite difference approximation to equations (2.59)-(2.62) using central difference quotients and average about the mid point (ζ^i,$\eta_{j-1/2}$) to obtain:

$$\frac{f_j^i - f_{j-1}^i}{h_j} = u_{j-1/2}^i, \quad \frac{U_j^i - U_{j+1}^i}{h_j} = V_{j+1/2}^i, \quad \frac{g_j^i - g_{j-1}^i}{h_j} = P_{j-1/2}^i \tag{22}$$

Similarly equations (13)-(16) can be expressed in finite difference form, by approximating the functions and their derivatives by central differences about the midpoints ($\xi^{ii-1/2}$,$\eta_{j-1/2}$), giving the following nonlinear difference equations:

$$\begin{aligned} &h_j^{-1}\left(\left|V_j\right|^{n-1}\left|V_j\right|^{n-1} - \left|V_{j-1}\right|^{n-1}V_{j-1}\right)^i + \alpha_1(fV)_{j-1/2}^i - \alpha_2\left(V^2\right)_{j-1/2}^i + g_{j-1/2}^i \\ &+ \alpha\left(V_{j-1/2}^{i-1}f_{j-1/2}^i - f_{j-1/2}^{i-1}V_{j-1/2}^i\right) = R_{j-1/2}^{i-1} \end{aligned} \tag{23}$$

$$PR(\xi)h_j^{-1}\left(P_j^i - P_{j-1}^i\right) + \alpha_1(fP)_{j-1/2}^i - \alpha\left[U_{j-1/2}^{i-1}g_{j-1/2}^i - U_{j-1/2}^i g_{j-1/2}^{i-1} + P_{j-1/2}^i f_{j-1/2}^{i-1} - P_{j-1/2}^{i-1} f_{j-1/2}^i\right] = X_{j-1/2}^{i-1} \tag{24}$$

where

$$R_{j-1/2}^{i-1} = -L_{j-1/2}^{i-1} + \alpha\left[(fV)_{j-1/2}^{i-1} - \left(V^2\right)_{j-1/2}^{i-1}\right] \tag{25}$$

$$\left[L_{j-1/2}^{i-1} = h_j^{-1}\left(\left|V_j\right|^{n-1}\left|V_j\right|^{n-1} - \left|V_{j-1}\right|^{n-1} V_{j-1}\right) + p_1(fV)_{j-1/2} - p_2\left(U^2\right)_{j-1/2} + \theta_{j-1/2}\right]^{i-1} \tag{26}$$

$$X_{j-1/2}^{i-1} = -Y_{j-1/2}^{i-1} + \alpha\left[(fP)_{j-1/2}^{i-1} - (Ug)_{j-1/2}^{i-1}\right] \tag{27}$$

$$Y_{j-1/2}^{i-1} = \left[h_j^{-1}\left(P_j - P_{j-1}\right) + p_1(fP)_{j-1/2}\right]^{i-1} \tag{28}$$

$$T_{j-1/2}^{i-1} = -M_{j-1/2}^{i-1} + \alpha\left[(fq)_{j-1/2}^{i-1} - (U\theta)_{j-1/2}^{i-1}\right] \tag{29}$$

$$M_{j-1/2}^{i-1} = \left[\frac{1}{\Pr}h_j^{-1}\left(q_j - q_{j-1}\right) + p_1(fq)_{j-1/2}\right]^{i-1} \tag{30}$$

$$\alpha = p_3\, k_i^{-1}\zeta^{i-1},\ \alpha_1 = p_1^i + \alpha,\ \alpha_2 = p_2^i + \alpha \tag{31}$$

The wall and the edge boundary conditions are

$$f_0^i = 0,\quad V_0^i = p_3(\xi)(U_0^i)^m = 0,\quad g_0^i = 1 \quad \text{and} \quad U_j^i = 0,\quad g_j^i == 0 \tag{32}$$

If we assume $f_j^{i-1}, U_j^{i-1}, V_j^{i-1}, g_j^{i-1}\, P_j^{i-1}$ to be known for $0 \leq j \leq J$, equations (22)-(24) are a system of $5J + 5$ nonlinear algebraic equations for $5J + 5$ unknowns (f_j^i, U_j^i, V_j^i, g_j^i, P_j^i), $j = 1, 2, \ldots, J$. These nonlinear system of algebraic equations are linearized by means of Newton's method and solved in a very efficient manner by using the Keller-box method, as discussed by Cebeci and Bradshaw (1984) in a simpler way. For a given ξ, the iterative procedure was stopped to give the final velocity and temperature distribution when the diference in computing these functions in the next procedure becomes less than 10^{-6}, i.e. $|\delta f^k| \leq 10^{-6}$, where the superscript k denotes iteration number. For these computations, a nonuniform grid in the η direction has been used, with $\eta_j = \sinh((j - 1)/a)$, where $j = 1, 2, 3, \ldots, J$. Here, $J = 601$ and $a = 250$ had been chosen in order to obtain quick convergence and thus to save computational time and space. It should be mentioned that convergent solutions at every ξ stations had been found within three iterations only. In the present integration scheme, values of ξ are increased with the increament $\Delta\zeta = 0.005$ for every variation of the pertinent parameters, such as n and λ for $Pr = 10.0$ and 100.0. Since a singular point at $\xi = 0$ exists in equation (8), we begin the integration with $\xi = 10^{-8}$.

Results and discussion

To confirm the accuracy of the present integration, we have compared in Table I the numerical results obtained by Acrivos (1960), Kawase and Ulbrecht (1984) and Huang *et al.* (1989) for the case of a rigid vertical surface.

The results of Acrivos (1960) and Kawase and Ulbrecht (1984) were based on an approximate analysis. The agreement between our results and those of Huang *et al.* (1989) is within 1 percent error.

Figures 2-4 display results for friction factor and Nusselt number. Figure 2 indicates that as the slip parameter λ increases, friction factor increases and the heat transfer rate decreases. Figure 3 indicates that the power-law index parameter n increases both the friction factor and heat transfer rate. From Figure 4, we notice that as the Prandtl number for the fluid increases, the friction factor decreases and the heat transfer rate increases.

Effects of the physical parameters n, λ, and ξ on the velocity and temperature fields need to be discussed. In Figures 5 and 6 the velocity and temperature profiles are shown graphically for values of the local non-similarity parameter ξ in the interval $[0, \infty]$ for the fluid having the value of the Prandtl number $Pr = 100$. For $Pr = 100$, the velocity profiles and the temperature profiles are displayed in Figures 5(a) and 5(b),

Table I.
Comparison of local Nusselt $Nu_x/Gr_x^{1/2(1+n)}$ number while $\xi = 1.0$, $Pr = 100$

n	Acrivos (1960)	Kawase and Ulbrecht (1984)	Huang *et al.* (1989)	Present
0.6	1.3375	1.3447	1.2754	1.26181
0.8	1.4788	1.4818	1.4334	1.42717
1.0	1.5903	1.5891	1.5486	1.54795
1.2	1.6930	1.6755	1.6408	1.64058

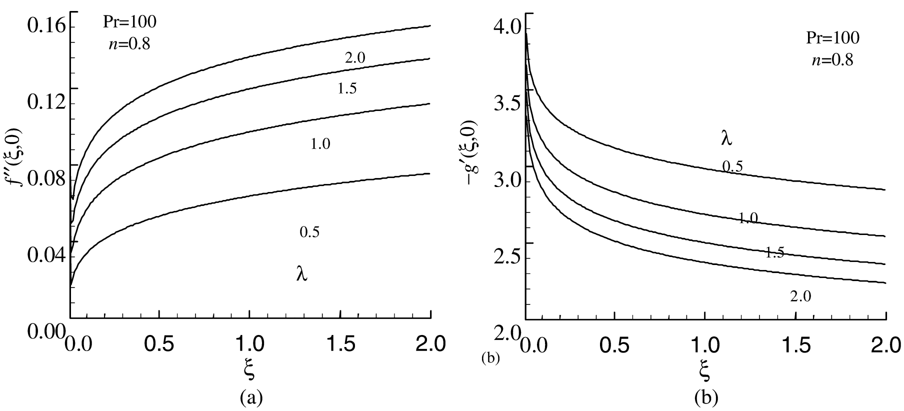

Figure 2.
(a) Shear-sress and (b) rate of heat transfer coefficient for different λ while $n = 0.8$ and $Pr = 100$

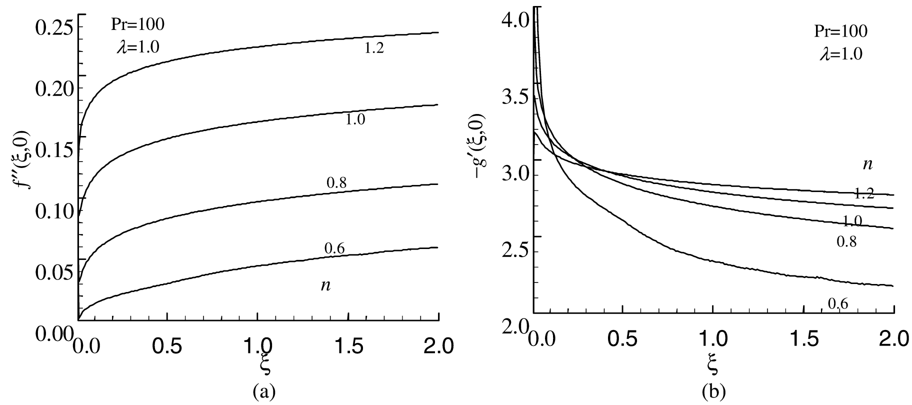

Figure 3.
(a) Shear-sress and (b) rate of heat transfer coefficient for different n while $\lambda = 1.0$ and $\Pr = 100$

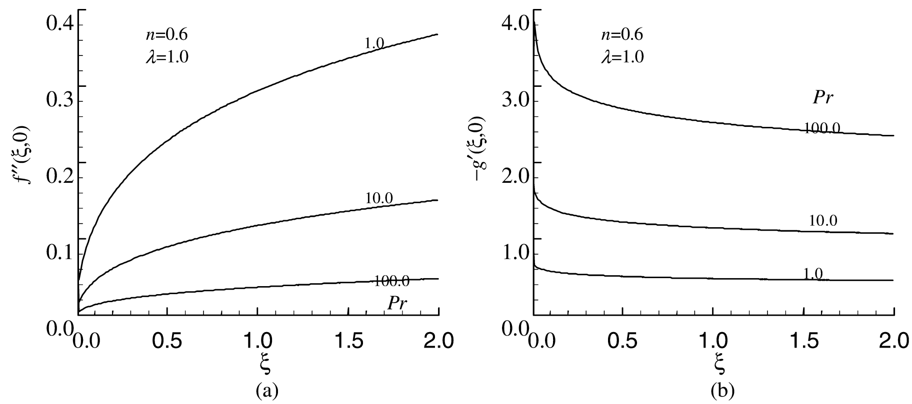

Figure 4.
(a) Shear-sress and (b) rate of heat transfer coefficient for different Pr while $\lambda = 1.0$ and $n = 0.6$

respectively, for different values of ξ considering the flow along an isothermal surface. From this figure, we can see that the velocity field decreases near the surface owing to increase of ξ and finally it leads towards it asymptotic profile while the flow is totally dominated by the buoyancy force. On the other hand, we see that velocity profiles overshoot at a point approximately $\eta = 1$ and then increase with the decrease of ξ; finally this also increases the boundary layer thickness. From Figure 5(b) we can further see that, increase in the value of the parameter ξ leads to increase in the temperature profile in the boundary layer region and also leads to increase in the thermal boundary layer thickness. From Figures 6(a) and 6(b), it may be noticed that as the slip parameter λ increases, the velocity maximum within the boundary layer increases and the temperature within the boundary layer increases. As λ increases, both the momentum and thermal boundary layer thicknesses increase.

Conclusions

In the present paper we have investigated the natural convection flow of a power-law type non-Newtonian fluid past a slotted vertical surface. The governing equations for the flow

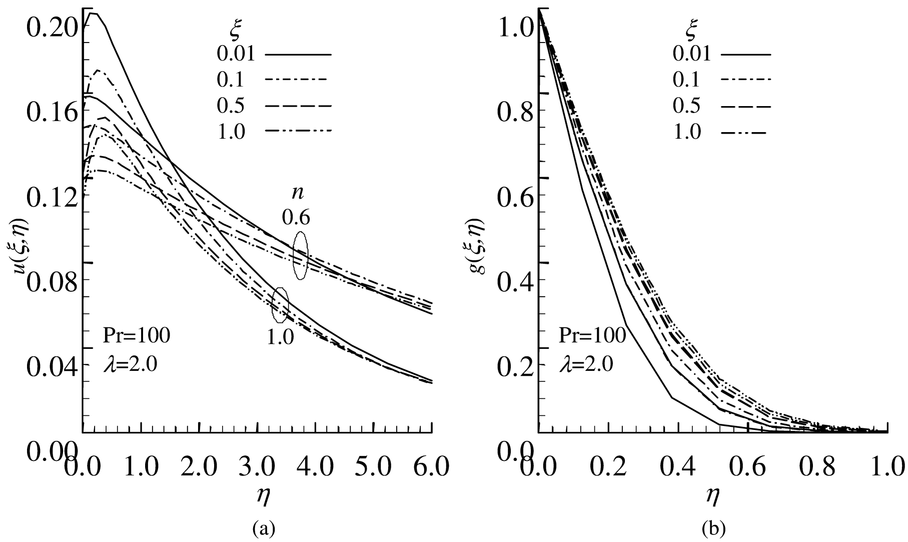

Figure 5.
(a) Velocity and (b) temperature profiles for different values of ξ against η for fluid having $n = 0.6$ and 1.0 while $\mathrm{Pr} = 100$ and $\lambda = 2.0$

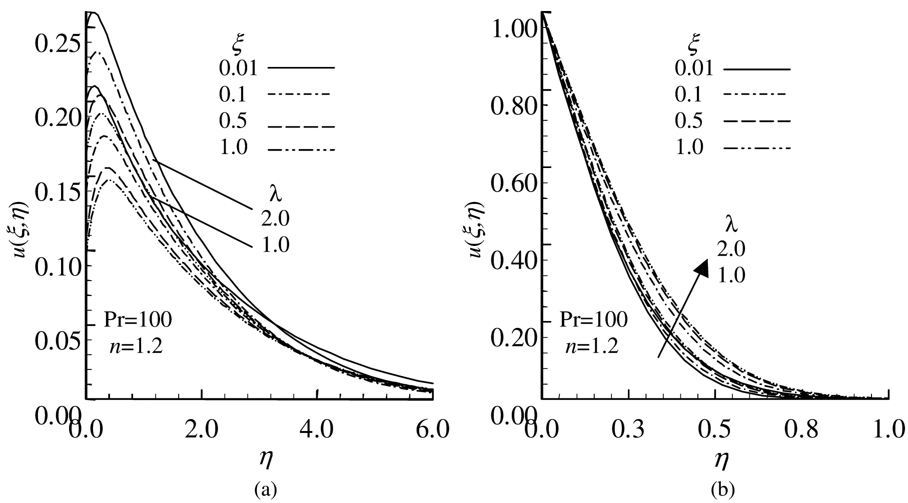

Figure 6.
(a) Velocity and (b) temperature profiles for different values of ξ against η for fluid having $n = 1.2$, $\mathrm{Pr} = 100$ while $\lambda = 1.0$ and 2.0

and heat transfer are reduced to local non-similarity equations, treating ξ as the local similarity variable, The transformed boundary-layer equations have been integrated numerically applying Keller-box method for all values of ξ in the interval $[0.0, \infty]$. The results are expressed in terms of the reduced local skin-friction factor, and local rate of heat transfer coefficients against ξ for varying values of n, λ, and Prandtl number, Pr.

The following conclusions can be obtained from the present investigation:

(1) Increase in ξ leads to increase in the values of the skin-friction coefficient, $F''(0, \xi)$ and decrease in the heat transfer rate.

(2) Owing to increase in the value the Prandtl number, Pr, there is decrease in the value of the skin-friction coefficient, $F''(0, \xi)$, and augmentation of heat transfer rate $-G'(0, \xi)$.

(3) As the viscosity index n increases, both the friction factor and the heat transfer rate increase.

(4) As the slip parameter λ increases, the friction factor increases whereas the heat transfer rate decreases.

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